

# Constrained Kalman Filtering Via Density Function Truncation for Turbofan Engine Health Estimation

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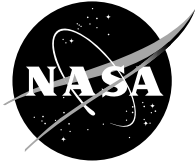
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## **Abstract**

Kalman filters are often used to estimate the state variables of a dynamic system. However, in the application of Kalman filters some known signal information is often either ignored or dealt with heuristically. For instance, state variable constraints (which may be based on physical considerations) are often neglected because they do not fit easily into the structure of the Kalman filter. This paper develops an analytic method of incorporating state variable inequality constraints in the Kalman filter. The resultant filter truncates the PDF (probability density function) of the Kalman filter estimate at the known constraints and then computes the constrained filter estimate as the mean of the truncated PDF. The incorporation of state variable constraints increases the computational effort of the filter but significantly improves

its estimation accuracy. The improvement is demonstrated via simulation results obtained from a turbofan engine model. The turbofan engine model contains 3 state variables, 11 measurements, and 10 component health parameters. It is also shown that the truncated Kalman filter may be a more accurate way of incorporating inequality constraints than other constrained filters (e.g., the projection approach to constrained filtering).

*Key Words* – Kalman Filter, State Constraints, Estimation, Probability Density Function, Gas Turbine Engines.

## 1 Introduction

For linear dynamic systems with white process and measurement noise, the Kalman filter is known to be an optimal estimator. However, in the application of Kalman filters there is often known model or signal information that is either ignored or dealt with heuristically [13]. This has resulted in recent efforts to incorporate constraints in the Kalman filter. For example, a projection method can be used to find the optimal way to incorporate hard inequality constraints on the states [20, 21]. Another way of incorporating constraints is to use a regularization method to enforce a soft limit on the changes of the state variables with respect to time [22]. Yet another approach is the use of ridge regression to bias estimates with low certainty toward their constraints [5].

This paper presents a way to generalize the Kalman filter in such a way that known inequality constraints among the state variables are satisfied by the state variable estimates. The constraints that are imposed are hard constraints in that they are strictly enforced. However, in contrast to the projection method of constraint enforcement [20, 21], the state estimates are not projected onto the constraint surface. Rather, the PDF that is computed by the Kalman filter is truncated at the constraint edges, and the constrained state estimate becomes equal to the mean of the truncated PDF. This idea is based on a previously published method [18] but has been modified to handle two-sided inequality constraints.

The application considered in this paper is turbofan engine health parameter

estimation [6]. The performance of gas turbine engines deteriorates over time. This deterioration reduces the fuel economy of the engine. Airlines periodically collect engine data in order to evaluate the health of the engine and its components. The health evaluation is then used to determine maintenance schedules. Reliable health evaluations are used to anticipate future maintenance needs. This offers the benefits of improved safety and reduced operating costs. The money-saving potential of such health evaluations is substantial, but only if the evaluations are reliable. The data used to perform health evaluations are typically collected during flight and later transferred to ground-based computers for post-flight analysis. Data are collected each flight at the same engine operating point and corrected to account for variability in ambient conditions. Typically, data are collected for a period of about 3 seconds at a rate of about 10 Hz. Various algorithms have been proposed to estimate engine health parameters, such as weighted least squares [7], expert systems [4], Kalman filters [25], neural networks [25], and genetic algorithms [11].

This paper develops the truncation method of constrained Kalman filtering, and then applies it to the estimation of engine component efficiencies and flow capacities. Engine component efficiencies and flow capacities are referred to as *health parameters*. We can use our knowledge of the physics of the turbofan engine in order to obtain a dynamic model [2, 24]. The health parameters that we try to estimate can be modeled as slowly varying biases. The state vector of the dynamic model is augmented to include the health parameters, which are then estimated with a Kalman filter [8]. We use heuristic knowledge of the health parameter dynamics to constrain their estimate. For example, we know that health parameters never improve. Engine health always degrades over time, and we can incorporate this information into state constraints to improve our health parameter estimation. (This is assuming that no maintenance or engine overhaul is performed.) It should be emphasized that in this paper we are confining the problem to the estimation of engine health parameters in the presence of degradation only. There are specific engine cases that can result in abrupt shifts in filter estimates, possibly even indicating an apparent improvement in some engine components. An actual engine performance monitoring system would need to include additional logic to detect and isolate such faults.

Section 2 derives the constrained Kalman filter. Section 3 discusses the problem of turbofan health parameter estimation, along with the dynamic model that we use in our simulation experiments. Although the health parameters are not state variables of the model, it is shown how the dynamic model can be augmented in such a way that a Kalman filter can estimate the health parameters [8, 12]. We then show how this problem can be expressed in a way that is compatible with the constraints discussed in the earlier section. Section 4 presents some simulation results based on a turbofan model linearized around a known operating point. We show that the truncated Kalman filter can estimate health parameters better than the unconstrained filter, and it can also estimate health parameters better than other constrained filters. Section 5 presents some concluding remarks and suggestions for further work.

## 2 Constrained Kalman Filtering

Consider the discrete linear time-invariant system given by

$$\begin{aligned} x(k+1) &= Ax(k) + w(k) \\ y(k) &= Cx(k) + e(k) \end{aligned} \tag{1}$$

where  $k$  is the time index,  $x$  is the state vector, and  $y$  is the measurement. The signals  $\{w(k)\}$  and  $\{e(k)\}$  are uncorrelated zero mean Gaussian noise input sequences with covariances

$$\begin{aligned} E[w(k)w^T(m)] &= Q\delta_{km} \\ E[e(k)e^T(m)] &= R\delta_{km} \\ E[w(k)e^T(m)] &= 0 \end{aligned}$$

where  $E[\cdot]$  is the expectation operator and  $\delta_{km}$  is the Kronecker delta function ( $\delta_{km} = 1$  if  $k = m$ ,  $\delta_{km} = 0$  otherwise). The Kalman filter equations are given as follows [1].

$$K(k) = A\Sigma(k)C^T(C\Sigma(k)C^T + R)^{-1} \tag{2}$$



$$\begin{aligned}\hat{x}(k+1) &= A\hat{x}(k) + Bu(k) + K(k)(y(k) - C\hat{x}(k)) \\ \Sigma(k+1) &= (A\Sigma(k) - K(k)C\Sigma(k))A^T + Q\end{aligned}$$

where the filter is initialized with  $\hat{x}(0) = E[x(0)]$ , and  $\Sigma(0) = E[(x - x(0))(x - x(0))^T]$ . The Kalman filter estimate  $\hat{x}(k)$  is a Gaussian random variable with a mean of  $x(k)$  and a covariance matrix of  $\Sigma(k)$ .

Now suppose that we are given the  $s$  scalar constraints

$$a_i(k) \leq \phi_i^T(k)x(k) \leq b_i(k) \quad i = 1, \dots, s \quad (3)$$

where  $a_i(k) < b_i(k)$ . This is a two sided constraint on some linear function of the state. If we have only a one sided constraint, then we set  $a_i(k) = -\infty$  or  $b_i(k) = \infty$ . Now suppose at time  $k$  that we have some estimate  $\hat{x}(k)$  with covariance  $\Sigma(k)$ . The problem is to truncate the Gaussian PDF  $N(x(k), \Sigma(k))$  at the  $s$  constraints given in (3), and then find the mean  $\tilde{x}(k)$  and covariance  $\tilde{\Sigma}(k)$  of the truncated PDF. These new quantities,  $\tilde{x}(k)$  and  $\tilde{\Sigma}(k)$ , become the constrained state estimate and its covariance.

In order to make the problem tractable, we will define  $\tilde{x}_i(k)$  as the state estimate after the first  $i$  constraints of (3) have been enforced, and  $\tilde{\Sigma}_i(k)$  as the covariance of  $\tilde{x}_i(k)$ . We therefore initialize

$$\begin{aligned}i &= 0 \\ \tilde{x}_i(k) &= \hat{x}(k) \\ \tilde{\Sigma}_i(k) &= \Sigma(k)\end{aligned} \quad (4)$$

Now perform the following transformation.

$$z_i(k) = RW^{-1/2}T^T(x(k) - \tilde{x}_i(k)) \quad (5)$$

where  $T$  and  $W$  are obtained from the Jordan canonical decomposition of  $\tilde{\Sigma}_i(k)$ .

$$TWT^T = \tilde{\Sigma}_i(k) \quad (6)$$

We see that  $T$  is orthogonal and  $W$  is diagonal (therefore its square root is very easy to compute). Note that  $z_i(k)$  has a mean of 0 and covariance matrix of identity.

Next we use Gram-Schmidt orthogonalization to find the orthogonal  $R$  that satisfies

$$RW^{1/2}T^T\phi_i(k) = \begin{bmatrix} (\phi_i^T(k)\tilde{\Sigma}_i(k)\phi_i(k))^{1/2} & 0 & \cdots & 0 \end{bmatrix} \quad (7)$$

With these definitions we see that the upper bound (3) is transformed as

$$\begin{aligned} \phi_i^T(k)x(k) &\leq b_i(k) \\ \phi_i^T(k)TW^{1/2}R^Tz_i(k) + \phi_i^T(k)\tilde{x}_i(k) &\leq b_i(k) \\ \frac{(\phi_i^T(k)TW^{1/2}R^T)z_i(k)}{(\phi_i^T(k)\tilde{\Sigma}_i(k)\phi_i(k))^{1/2}} &\leq \frac{b_i(k) - \phi_i(k)^T\tilde{x}_i(k)}{(\phi_i^T(k)\tilde{\Sigma}_i(k)\phi_i(k))^{1/2}} \\ \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} z_i(k) &\leq \frac{b_i(k) - \phi_i(k)^T\tilde{x}_i(k)}{(\phi_i^T(k)\tilde{\Sigma}_i(k)\phi_i(k))^{1/2}} \\ &\leq d_i(k) \end{aligned} \quad (8)$$

where  $d_i(k)$  is defined by the above equation. Similarly we can see that

$$\begin{aligned} \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} z_i(k) &\geq \frac{a_i(k) - \phi_i(k)^T\tilde{x}_i(k)}{(\phi_i^T(k)\tilde{\Sigma}_i(k)\phi_i(k))^{1/2}} \\ &\geq c_i(k) \end{aligned}$$

where  $c_i(k)$  is defined by the above equation. We therefore have the normalized scalar constraint

$$c_i(k) \leq \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} z_i(k) \leq d_i(k) \quad (9)$$

Since  $z_i(k)$  has a covariance of identity, its elements are statistically independent of each other. Only the first element of  $z_i(k)$  is constrained, so the PDF truncation reduces to a one dimensional PDF truncation. The first element of  $z_i(k)$  is distributed as  $N(0,1)$  (before constraint enforcement), but the constraint says that  $z_i(k)$  must lie between  $c_i(k)$  and  $d_i(k)$ . We therefore remove that part of the Gaussian PDF that is outside of the constraints and compute the area of the remaining portion of the PDF as

$$\int_{c_i(k)}^{d_i(k)} \frac{1}{\sqrt{2\pi}} \exp(-\zeta^2/2) d\zeta = \frac{1}{2} \left[ \text{erf}(d_i(k)/\sqrt{2}) - \text{erf}(c_i(k)/\sqrt{2}) \right] \quad (10)$$

where  $\text{erf}(\cdot)$  is the error function, defined as

$$\text{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t \exp(-t^2/2) dt \quad (11)$$

We normalize the truncated PDF so that it has an area of one, and we find that the truncated PDF (i.e., the constrained PDF of the first element of  $z_i(k)$ ) is given by

$$\begin{aligned} \text{PDF}(\zeta) &= \alpha \exp(-\zeta^2/2) \\ \alpha &= \frac{\sqrt{2}}{\sqrt{\pi} [\text{erf}(d_i(k)/\sqrt{2}) - \text{erf}(c_i(k)/\sqrt{2})]} \end{aligned} \quad (12)$$

We can compute the mean and variance of  $z_i(k)$  as

$$\begin{aligned} \mu &= E[z_i(k)] \\ &= \alpha \int_{c_i(k)}^{d_i(k)} \zeta \exp(-\zeta^2/2) d\zeta \\ &= \alpha [\exp(-c_i^2(k)/2) - \exp(-d_i^2(k)/2)] \\ \sigma^2 &= E[(z_i(k) - \mu)^2] \\ &= \alpha \int_{c_i(k)}^{d_i(k)} (\zeta - \mu)^2 \exp(-\zeta^2/2) d\zeta \\ &= \alpha [\exp(-c_i^2(k)/2)(c_i - 2\mu) - \exp(-d_i^2(k)/2)(d_i - 2\mu)] + \mu^2 + 1 \end{aligned} \quad (13)$$

The mean and variance of the transformed state estimate, after enforcement of the first constraint, are therefore given as

$$\begin{aligned} \tilde{z}_{i+1}(k) &= \begin{bmatrix} \mu & 0 & \cdots & 0 \end{bmatrix} \\ \text{Cov}(\tilde{z}_{i+1}(k)) &= \text{diag}(\sigma^2, 1, \dots, 1) \end{aligned} \quad (14)$$

We then take the inverse of the transformation (5) to find the mean and variance of the state estimate after enforcement of the first constraint.

$$\begin{aligned} \tilde{x}_{i+1}(k) &= TW^{1/2}R^T \tilde{z}_{i+1}(k) + \tilde{x}_i(k) \\ \tilde{\Sigma}_{i+1}(k) &= TW^{1/2}R^T \text{Cov}(\tilde{z}_{i+1}(k))RW^{1/2}T^T \end{aligned} \quad (15)$$

We then increment  $i$  by one and repeat the process of (5)–(15) to obtain the state estimate after enforcement of the next constraint. After going through this process  $s$  times (once for each constraint) we have the final constrained state estimate and covariance at time  $k$ .

$$\begin{aligned} \tilde{x}(k) &= \tilde{x}_s(k) \\ \tilde{\Sigma}(k) &= \tilde{\Sigma}_s(k) \end{aligned} \quad (16)$$

Figure 1 shows an example of a one-dimensional state estimate before and after truncation. Before truncation the state estimate is outside of the state constraints. After truncation, the state estimate is set equal to the mean of the truncated PDF. Figure 2 shows another example. In this case the unconstrained state estimate is inside the state constraints. However, truncation changes the PDF and so the constrained state estimate changes to the mean of the truncated PDF.

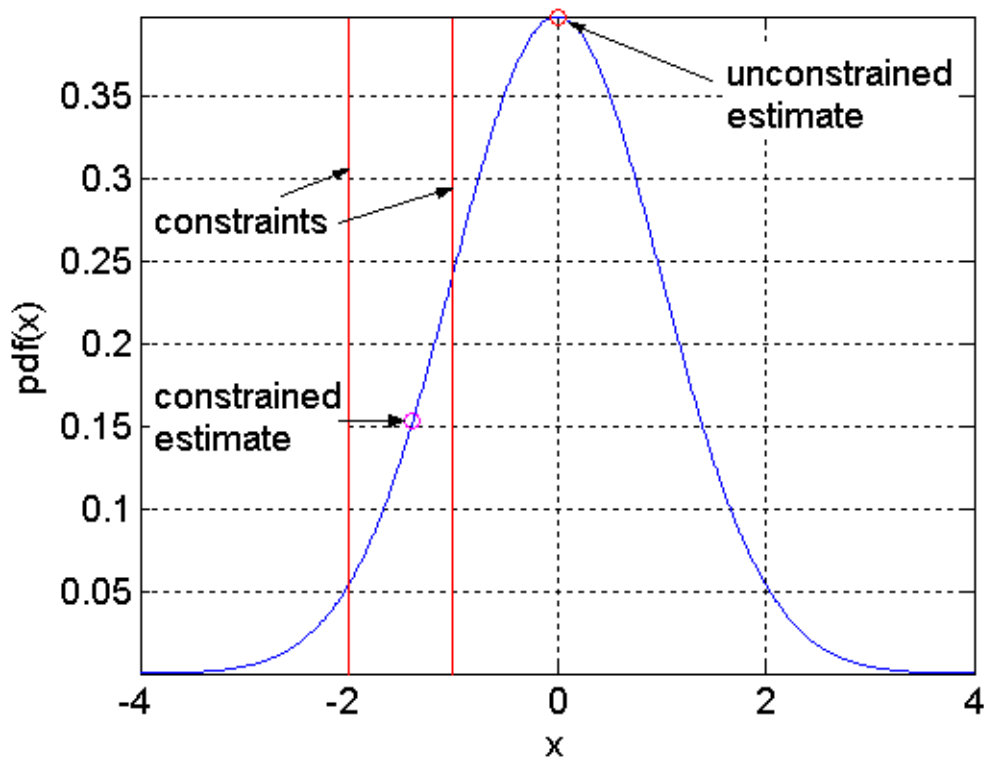


Figure 1: The unconstrained estimate violates the constraints. The constrained estimate is the centroid of the truncated PDF.

### 3 Turbofan Engine Health Monitoring

Figure 3 shows a schematic representation of a turbofan engine [16]. A single inlet supplies airflow to the fan. Air leaving the fan separates into two streams: one stream passes through the engine core, and the other stream passes through the

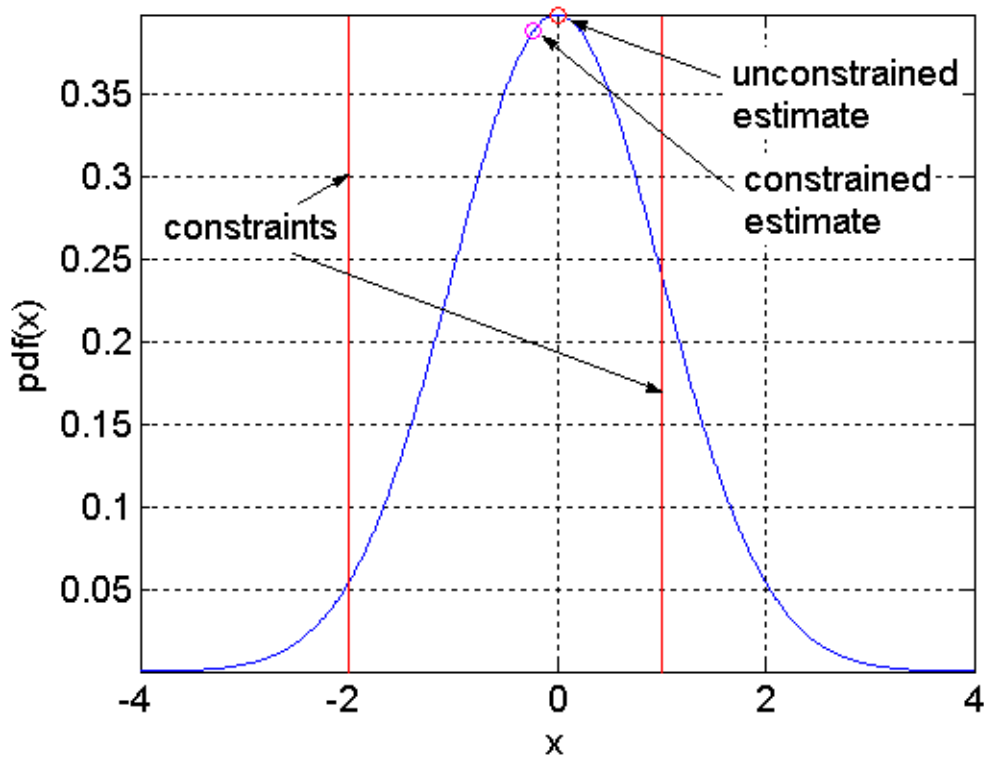


Figure 2: The unconstrained estimate satisfies the constraints. Nevertheless, the truncation approach to constrained estimation shifts the estimate to the centroid of the truncated PDF.

annular bypass duct. The fan is driven by the low pressure turbine. The air passing through the engine core moves through the compressor, which is driven by the high pressure turbine. Fuel is injected in the main combustor and burned to produce hot gas for driving the turbines. The two air streams combine in the augmentor duct, where additional fuel is added to further increase the air temperature. The air leaves the augmentor through the nozzle, which has a variable cross section area.

Various turbofan simulation packages have been developed over the years [2, 3, 10, 15]. The simulation used in this paper is a gas turbine engine simulation software package called MAPSS (Modular Aero Propulsion System Simulation) [16]. MAPSS is written using Matlab Simulink. The MAPSS engine model is based on a low frequency, transient, performance model of a high-pressure ratio, dual-spool,

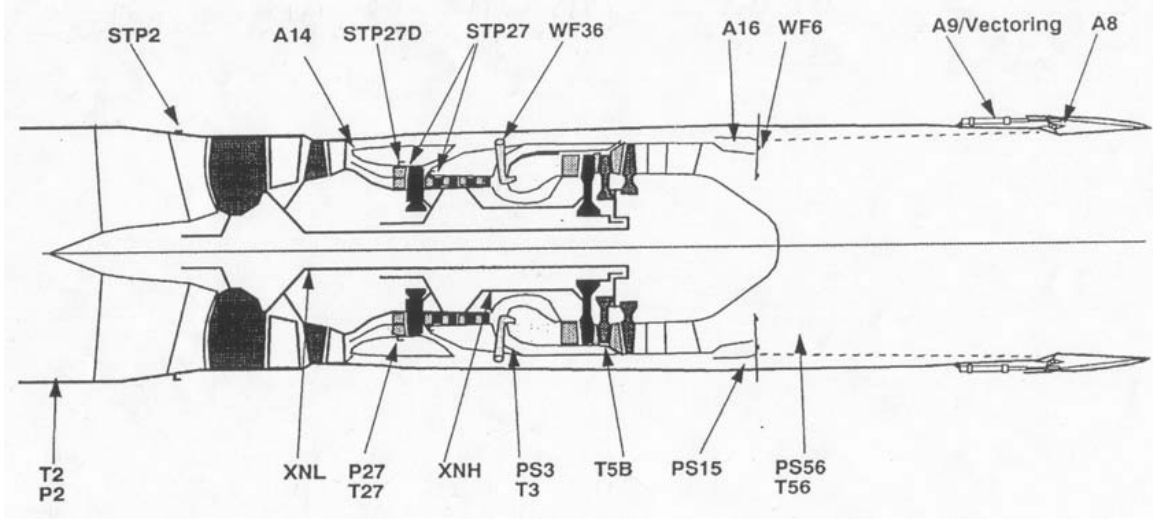


Figure 3: Schematic representation of a turbofan engine.

low-bypass, military-type, variable cycle, turbofan engine with a digital controller. The controller update rate is 50 Hz, and the component level model simulates the dynamics of the engine components at a rate of 2500 Hz. The three state variables used in MAPSS are low-pressure rotor speed (XNL), high-pressure rotor speed (XNH), and the average hot section metal temperature (TMPC) (measured from aft of the combustor to the high pressure turbine). The discretized time invariant equations that model the turbofan engine can be summarized as follows.

$$\begin{aligned}
 x(k+1) &= f[x(k), u(k), p(k)] + w_x(k) \\
 p(k+1) &= p(k) + w_p(k) \\
 y(k) &= g[x(k), u(k), p(k)] + e(k)
 \end{aligned} \tag{17}$$

where  $k$  is the time index,  $x$  is the 3-element state vector,  $u$  is the 3-element control vector,  $p$  is the 10-element health parameter vector, and  $y$  is the 11-element measurement vector. The health parameters change slowly over time. Between measurement times their deviations can be approximated by the zero mean noise  $w_p(k)$ . The noise term  $w_x(k)$  represents inaccuracies in the system model, and  $e(k)$  represents measurement noise. An extended Kalman filter can be used with (17) to estimate the state vector  $x$  and the health parameter vector  $p$ .

The states, controls, health parameters, and measurements are summarized in Tables 1–4, along with their values at the nominal operating point considered in this paper (a power lever angle of  $21^\circ$  at zero speed at sea level). Table 4 also shows typical signal-to-noise ratios for the measurements, based on NASA experience and previously published data [14]. Sensor dynamics are assumed to be high enough bandwidth that they can be ignored in the dynamic equations. In Tables 1–4, *LPT* is used for *Low Pressure Turbine*, *HPT* is used for *High Pressure Turbine*, *LPC* is used for *Low Pressure Compressor*, and *HPC* is used for *High Pressure Compressor*.

State	Nominal Value
LPT Rotor Speed	7 264 RPM
HPT Rotor Speed	12 152 RPM
Average Hot Section Metal Temperature	1 533 °R

Table 1: MAPSS turbofan model states and nominal values.

Control	Nominal Value
Main Burner Fuel Flow	2 454 lbm / hr
Variable Nozzle Area	343 in <sup>2</sup>
Rear Bypass Door Variable Area	154 in <sup>2</sup>

Table 2: MAPSS turbofan model controls and nominal values.

Constraints can be incorporated in the state estimator by using heuristic knowledge of the behavior of the health parameters. For example, it is known that health parameters never improve with time. It is also known that they degrade within a specific envelope.

$$\begin{aligned}
p_m(k) &\leq p_m^{\max}(k+1), & m \in [1-10] \\
p_m(k) &\geq p_m^{\min}(k+1)
\end{aligned} \tag{18}$$

This envelope constraint is in the linear form required in the constrained filtering problem statement (3) and is therefore amenable to the approach presented in this

Health Parameter	Nominal Value
Fan airflow	1
Fan efficiency	1
Booster tip airflow	1
Booster tip efficiency*	1
Booster hub airflow	1
Booster hub efficiency	1
High pressure turbine airflow	1
High pressure turbine efficiency	1
Low pressure turbine airflow	1
Low pressure turbine efficiency	1

Table 3: MAPSS turbofan model health parameters and nominal values.  
 (\*) The fourth health parameter is not yet implemented in MAPSS.

paper. Note that this does not take into account the possibility of abrupt changes in health parameters due to discrete damage events. That possibility must be addressed by some other means (e.g., residual checking [6]) in conjunction with the methods presented in this paper.

## 4 Simulation Results

We simulated the methods discussed in this paper using Matlab. We measured a steady state 3 second burst of engine data at 10 Hz during each flight. Each of these routine data collections was performed at the single operating point shown in Tables 1–4, except the engine’s health parameters deteriorated a small amount each flight. The signal-to-noise ratios were determined on the basis of NASA experience and previously published data [14] and are shown in Table 4. The models on which this work was based are fairly comprehensive, so we assumed that the process noise for each component of the state derivative equation (17) was zero. However, in the Kalman filter we used a one-sigma state process noise equal to 0.005% of the nominal state values to allow the filter to be responsive to changes in the state variables. We also set the one sigma process noise for each component of the health parameter



Measurement	Nominal Value	SNR
LPT exit pressure	19.33 psia	100
LPT exit temperature	1394 °R	100
Percent low pressure spool rotor speed	63.47%	150
HPC inlet temperature	580.8 °R	100
HPC exit temperature	965.1 °R	200
Bypass duct pressure	20.66 psia	100
Fan exit pressure	17.78 psia	200
Booster inlet pressure	20.19 psia	200
HPC exit pressure	85.06 psia	100
Core rotor speed	12 152 RPM	150
LPT blade temperature	1179 °R	70

Table 4: MAPSS turbofan model measurements, nominal values, and signal-to-noise ratios.

to 0.01% of the nominal parameter value. These values were obtained by tuning. They were small enough to give reasonably smooth estimates, and large enough to allow the filter to track slowly time-varying parameters. In the enforcement of the constraints in (18) we chose the constraint envelope as follows.

1. For the turbine airflow health parameters ( $m \in [7, 9]$ ), whose values *increase* with time (i.e., an increase corresponds to a degradation),  $p_m^{\max}(k)$  was set equal to a linear-plus-exponential degradation that was initialized to zero (i.e.,  $p_m^{\max}(0) = 0$ ) and reached a maximum of 6% after 500 flights, while  $p_m^{\min}(k)$  was set equal to 0 for all  $k$ .
2. For the other health parameters ( $m \in [1 - 6, 8, 10]$ ), whose values *decrease* with time (i.e., a decrease corresponds to a degradation),  $p_m^{\min}(k)$  was set equal to a linear-plus-exponential degradation that was initialized to zero (i.e.,  $p_m^{\min}(0) = 0$ ) and reached a maximum magnitude of -6% after 500 flights, while  $p_m^{\max}(k)$  was set equal to 0 for all  $k$ .

We simulated a linear-plus-exponential degradation of the 10 health parameters over 100 flights. The initial health parameter estimation errors were assumed to be

zero. The simulated health parameter degradations were representative of turbofan performance data reported in the literature [17].

Figure 4 shows a typical plot of the true deviation of health parameter 10, along with the constraint envelope, the unconstrained estimate, and the constrained estimate. It is seen that even though the unconstrained estimate lies within the constraint envelope, the constrained estimate is more accurate. Figure 5 shows a different type of example where the true health parameter deviation is closer to the constraint envelope. In this case there are times when the unconstrained estimate lies outside of the constraint envelope, but the enforcement of constraints forces the constrained estimate to remain within the envelope.

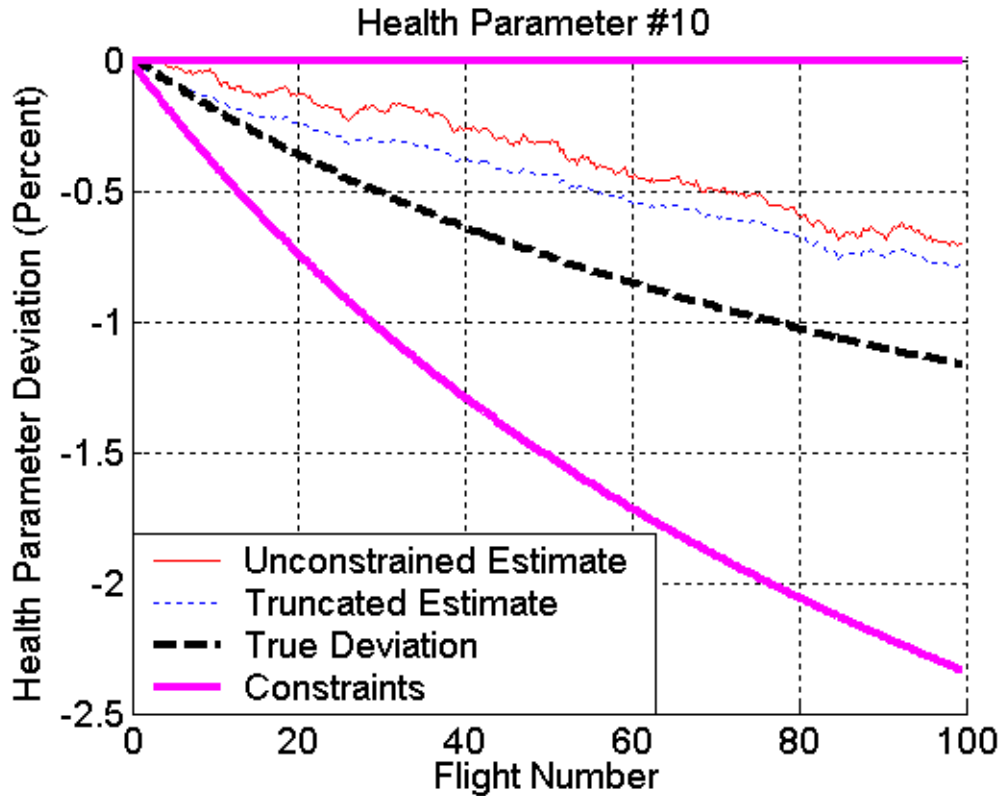


Figure 4: In this example, constraint enforcement decreases the RMS estimation error from 12.2% to 9.2%.

We ran 20 Monte Carlo simulations, each with a different noise history. We obtained estimates of the health parameters using three different methods.

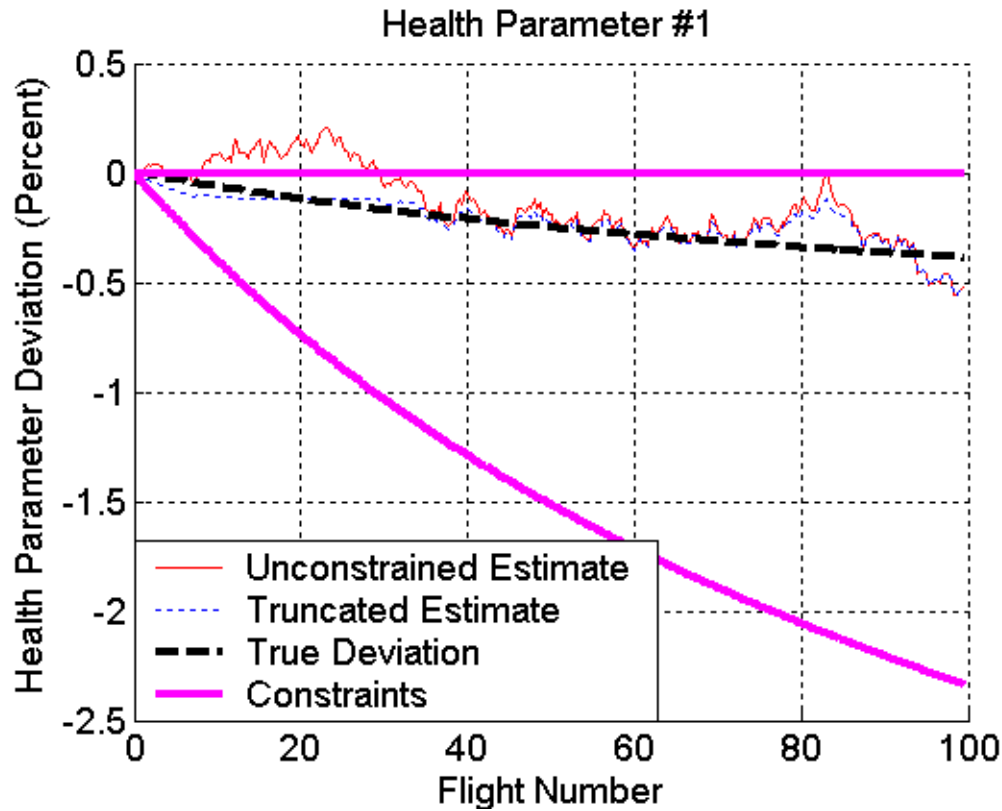


Figure 5: In this example, constraint enforcement decreases the RMS estimation error from 13.4% to 6.6%.

1. Unconstrained (standard) Kalman filtering.
2. Constrained Kalman filtering using the projection approach [20, 21].
3. Constrained Kalman filtering using the projection approach and constraint tuning [23].
4. Constrained Kalman filtering using the truncation approach discussed in this paper.

Table 5 shows the performance of the filters averaged over all 20 simulations. The standard Kalman filter estimates the health parameters to within 7.4% of their final degradations. The projection-based constrained filter estimates the health parameters to within 6.5% of their final degradations. The projection-based constrained

filter with the addition of residual-based tuning estimates the health parameters to within 6.1% of their final degradations. Finally, the use of the truncation approach for constrained filtering estimates the parameters to within 6.1% of their final degradations. These numbers show the improvement that is possible with the truncation approach to constrained Kalman filtering. Although we may be able to get just as good performance using the tuned projection filter, a lot more tuning is required than with the truncation approach [23].

Health Parameter	Estimation Error (%)			
	Unconstrained Filter	Projection Filter	Tuned Filter	Truncated Filter
Fan airflow	12.9	9.2	8.2	7.5
Booster hub airflow	6.9	6.2	6.0	5.8
Booster tip airflow	10.9	10.6	10.0	10.5
Booster tip efficiency*	N/A	N/A	N/A	N/A
Booster hub airflow	7.4	6.8	6.3	6.4
Booster hub efficiency	3.8	3.1	3.0	3.7
High pressure turbine airflow	4.3	3.3	3.2	4.0
High pressure turbine efficiency	4.2	3.8	3.7	4.2
Low pressure turbine airflow	3.6	3.3	3.2	3.8
Low pressure turbine efficiency	11.3	11.2	11.1	8.8
Average	7.4	6.5	6.1	6.1

Table 5: Health parameter estimation errors (percent) of the Kalman filters.  
(\*) The fourth health parameter is not yet implemented in MAPSS.

The improved performance of the constrained filter comes with a price, and that price is computational effort. The algorithm outlined in (5)–(15) requires Jordan decomposition and Gram-Schmidt orthogonalization. However, if the constraints of (3) are decoupled (as they are in our example) then the computational effort can be largely reduced by ignoring the cross-covariance terms in the state estimator and hence avoiding these matrix computations. In any case, computational effort is not a critical issue for turbofan health estimation since the filtering is performed on ground-based computers after each flight.

Note that the Kalman filter works well only if the assumed system model matches reality fairly closely. The method presented in this paper, by itself, will not work well if there are large sensor biases or hard faults due to severe component failures. A mission-critical implementation of a Kalman filter should always include some sort of additional residual check to verify the validity of the Kalman filter results [9], particularly for the application of turbofan engine health estimation considered in this paper [6].

## 5 Conclusion and Discussion

We have presented a PDF truncation based method for incorporating constraints into a Kalman filter. If the system whose state variables are being estimated has known state variable constraints, then those constraints can be incorporated into the Kalman filter as shown in this paper. For the aircraft turbofan engine health estimation problem, the use of constraints generally improves the accuracy of health estimation. At first this seems counterintuitive, since the unconstrained Kalman filter is by definition the minimum variance filter. However, we have changed the system by introducing state variable constraints. Therefore, the unconstrained Kalman filter is no longer the minimum variance filter, and we can do better with the constrained Kalman filter.

We have seen that the constrained filter requires more computational effort than the standard Kalman filter. This is due to the addition of  $s$  matrix decompositions that must be performed at each time step (one for each constraint). The engineer must therefore perform a tradeoff between computational effort and estimation accuracy. For real time applications the improved estimation accuracy may or may not be worth the increase in computational effort.

The Kalman filter works well only if the assumed system model matches reality fairly closely. The constraint enforcement and constraint tuning methods presented in this paper will not work well if there are large sensor biases or hard faults due to severe component failures. A mission-critical implementation of a Kalman filter should always include some sort of residual check to verify the validity of the Kalman

filter results, particularly for the application of turbofan engine health estimation considered in this paper [6, 9].

Although we have considered only linear state constraints, it is not conceptually difficult to extend this paper to nonlinear constraints. If the state constraints are nonlinear they can be linearized as discussed in [19]. Further work could explore ways to optimally tune the constraints of the truncated Kalman filter.

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